plasma oscillation & plasma Frequency

First treatment of plasma oscillation was given by two scientists Tonk and Langmuir. Tonk and Langmuir considered an idealised plasma in which there is no thermal motion of ions and electrons. For no thermal motion, $T=0 \Rightarrow KT=0$ where K is Boltzmann constant.

There are two types of oscillation in plasma

1. Electron ($\bar{e}$) oscillation 2. Ion oscillation

Electron oscillation is so fast that ions can be regarded as stationary in comparision with electrons. ion oscillation is slow due to its large inertia. Electrons adjust their energy and density to remain in equilibrium while ion oscillation is so slow that can be neglected and ions are only used to keep plasma neutral. Therefore, in plasma oscillation, oscillation of electrons are mainly used. i.e. plasma oscillation means electron oscillation.

If the electrons in a plasma are displaced from a uniform background of ions then electric field will be built up in such a direction as to restore the neutrality of the plasma by pulling the electrons back to their original positions. But because of their inertia, electrons will pass through their original positions and they begin to oscillate about their equilibrium positions with a characteristic frequency known as the plasma frequency. This oscillation is so fast that the massive ions do not have time to respond to the oscillating field and the massive ions may be considered as fixed.

In shown fig-1, the open circles represent typical elements of ion fluid and the darkened circles represent alternately displaced elements of electron fluid. The resulting charge bunching causes (produces) a periodic field E which tends to restore the electrons to their neutral (or equilibrium) positions.

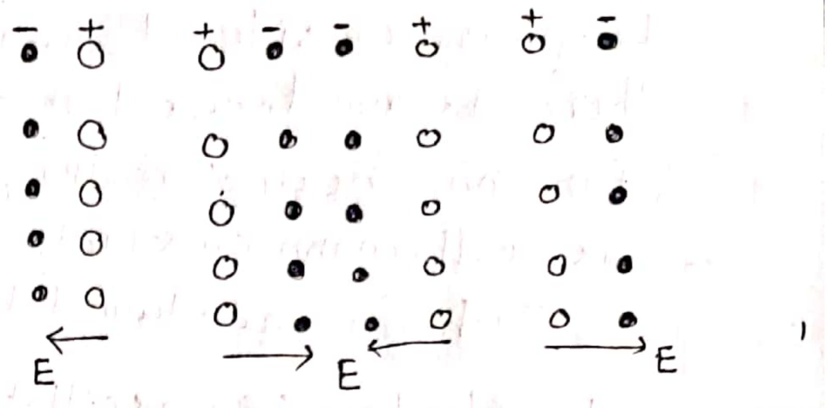

Fig-1: Mechanism of plasma oscillation

Now we are going to derive an expression for the plasma frequency ω_p in the simplest case by making the following assumptions.

(1) There is no magnetic field (2) there is no thermal motions ($KT=0$) ③ the ions are fixed in space in a uniform distribution ④ the plasma is infinite in extent.

Using these four assumptions, the expression for the plasma frequency ω_p is derived.

plasma oscillation is electrostatic oscillation because magnetic field $B=0$.

Equation of continuity is

$$\nabla\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0 \quad \text{———} \quad ①$$

Here $\vec{J}=-n_e.e.\vec{V_e}$ and $\rho=-n_e.e$

where n_e = electron density, $-e$ = charge of electron and $\vec{V_e}$ = Drift velocity of electrons.

Using these in equ ①, we get

$$\nabla\cdot(-n_e\cdot e\cdot\vec{V_e}) + \frac{\partial(-n_e\cdot e)}{\partial t} = 0$$

$$\Rightarrow \nabla\cdot(n_e\cdot\vec{V_e}) + \frac{\partial n_e}{\partial t} = 0 \quad \text{———} \quad (2)$$

For solving equ ②, we make it linear.

For making the equ ② linear, we neglect the term containing higher powers of amplitude. Dependent variables are expressed as the sum of its equilibrium value (represented by subscript '0') and the perturbed (disturbed) value (represented by subscript '1').

$$\text{So } n_e = n_0 + n_1, \; \vec{V_e} = \vec{V_0} + \vec{V_1} \; \& \; \vec{E} = \vec{E_0} + \vec{E_1} \quad \text{———} \quad (3)$$

Using equ③ in equ②, we get

$$\nabla\cdot\{(n_0+n_1)\cdot(\vec{V_0}+\vec{V_1})\} + \frac{\partial(n_0+n_1)}{\partial t} = 0 \quad \text{———} \quad (4)$$

As before disturbing the plasma medium i.e, before displacing electrons from their equilibrium positions in plasma, $\vec{V_0} = 0$, $\vec{E_0} = 0 \Rightarrow \frac{\partial\vec{V_0}}{\partial t} = \frac{\partial\vec{E_0}}{\partial t} = 0$

n_0 = constant $\Rightarrow \nabla n_0 = 0$ and $n_0 \neq n_0(\vec{r},t) \Rightarrow \frac{\partial n_0}{\partial t} = 0$

Using these in equ④, We get

$$\nabla\cdot\{(n_0+n_1)\cdot\vec{V_1}\} + \frac{\partial n_1}{\partial t} = 0 \Rightarrow \nabla\cdot(n_0\vec{V_1}) + \nabla\cdot(n_1\vec{V_1}) + \frac{\partial n_1}{\partial t} = 0$$

Since $\nabla\cdot(n_1\vec{V_1})$ is a quadratic term so to linearize the equ, it is neglected.

$$\text{Thus } \nabla\cdot(n_0\vec{V_1}) + \frac{\partial n_1}{\partial t} = 0 \quad \text{———} \quad (5)$$

Diff. equ⑤ partially w.r.t. time.

$$\frac{\partial}{\partial t}[\nabla\cdot(n_0\vec{V_1})] + \frac{\partial^2 n_1}{\partial t^2} = 0$$

$$\Rightarrow \nabla\cdot\frac{\partial}{\partial t}(n_0\vec{V_1}) + \frac{\partial^2 n_1}{\partial t^2} = 0$$

$$\Rightarrow n_0\cdot\nabla\cdot\left(\frac{\partial\vec{V_1}}{\partial t}\right) + \frac{\partial^2 n_1}{\partial t^2} = 0 \quad \text{———} \quad (6) \quad \because n_0 = \text{constant.}$$

Here $\frac{\partial \vec{v}_1}{\partial t} = \vec{a}$ = acceleration of electron.

Now $\frac{\partial \vec{v}_1}{dt} = \vec{a} = \frac{\vec{F}}{m} = -\frac{e\vec{E}_1}{m}$ put in eqⁿ ⑥, we get

$$n_0 \nabla\left(-\frac{e\vec{E}_1}{m}\right) + \frac{\partial^2 n_1}{\partial t^2} = 0 \Rightarrow -\frac{n_0 e}{m}.(\vec{\nabla}.\vec{E}_1) + \frac{\partial^2 n_1}{\partial t^2} = 0 \quad (7)$$

From Maxwell's eqⁿ, $\vec{\nabla}.\vec{E}_1 = \frac{\rho_1}{\varepsilon_0} = \frac{-n_1 e}{\varepsilon_0}$ put in eqⁿ ⑦

$$-\frac{n_0 e}{m}.\left(-\frac{n_1 e}{\varepsilon_0}\right) + \frac{\partial^2 n_1}{\partial t^2} = 0 \Rightarrow \frac{\partial^2 n_1}{\partial t^2} + \frac{n_0 e^2}{m\varepsilon_0}.n_1 = 0$$

$$\Rightarrow \frac{\partial^2 n_1}{\partial t^2} + \omega_p^2.n_1 = 0 \quad (8)$$

It is an eqⁿ of SHM so n_1 is oscillating as SHM

where $\omega_p^2 = \frac{n_0 e^2}{m\varepsilon_0} \Rightarrow \boxed{\omega_p = \sqrt{\frac{n_0 e^2}{m\varepsilon_0}}}$ — (9) It is the expression for plasma frequency.

Now $$\boxed{f_p = \frac{\omega_p}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{n_0 e^2}{m\varepsilon_0}}} \quad (10)$$

$e = 1.6\times10^{-19}C$, $m = 9.1\times10^{-31}kg$, $\varepsilon_0 = 8.85\times10^{-12} C^2/Nm^2$

$$f_p = \frac{1}{2\times3.14}\sqrt{\frac{n_0.\ 1.6\times10^{-19}\times1.6\times10^{-19}}{9.1\times10^{-31}\times8.85\times10^{-12}}} \Rightarrow f_p = 9\sqrt{n_0}$$

This frequency, depending only on average number density of plasma (n_0), is one of the fundamental parameter of plasma. Because of smallness of m, the plasma frequency f_p is usually very high.

For instance (example), $n_0 = 10^{18}\ m^{-3}$, we have

$$f_p = 9.\sqrt{10^{18}} \Rightarrow f_p = 9\times10^{9}Hz = 9\ GHz$$

Radiation at f_p normally lies in the microwave range.

Ques:- Explain plasma oscillation. Derive an expression for plasma frequency.